Vol. 19 (2026): Proceedings of 2026 International Conference on Financial Technology, Mathematical Statistics, and Business Analytics (ICFMB 2026)
E-ISSN: 3105-0522 P-ISSN: 3105-0514
DOI: https://doi.org/10.70267/icfmb.202619382386

A Theoretical and Literature-Based Comparison of the Markowitz Model and the Index Model

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Abstract

This paper presents a comprehensive review of two foundational portfolio models in modern finance: the Markowitz mean-variance model and the Sharpe single-index model. The study systematically examines their theoretical frameworks, computational requirements, empirical performance, and practical applicability. The Markowitz model, as a model-free optimisation framework, aims to achieve theoretically optimal risk-return trade-offs by fully considering pairwise covariances among assets. However, its high sensitivity to estimation errors and intensive data requirements often limit its empirical usefulness, particularly for large portfolios. In contrast, the index model simplifies covariance estimation by attributing asset return variations to a common market factor, thereby reducing parameter dimensionality and enhancing computational efficiency. Multi-factor extensions further improve explanatory power while maintaining practical feasibility. By reviewing existing literature and empirical evidence, this paper highlights the strengths, limitations, and appropriate application scenarios of both models. The findings provide guidance for investors, portfolio managers, and researchers in selecting or integrating portfolio models to balance theoretical rigor with empirical robustness and operational efficiency.

Keywords

markowitz model, index model, portfolio optimisation, mean-variance analysis, factor models

1. Introduction

The core tenet of Modern Portfolio Theory (MPT) centers on attaining the optimal trade-off between risk and return [1]. Within this research domain, the mean-variance model proposed by Harry Markowitz in 1952 and the single-index model introduced by William F. Sharpe in 1963 constitute two of the most canonical analytical frameworks. These models embody two distinct methodological paradigms in portfolio theory—comprehensive characterization versus structural simplification—exerting a profound and enduring influence on subsequent financial theory and practice. Specifically, the Markowitz model theoretically enables optimal portfolio allocation by comprehensively capturing the expected returns, variances, and pairwise covariances of assets. However, its high sensitivity to parameter estimation and susceptibility to the curse of dimensionality as the number of assets expands constrain its applicability in large-scale asset allocation contexts. In contrast, Sharpe's single-index model attributes the co-movement of asset returns to a common systematic market factor, thereby drastically reducing the complexity of parameter estimation and enhancing the model's operational feasibility and stability. Subsequently, the evolution of multi-factor models (e.g., the Fama-French model) has further strengthened the explanatory power of asset returns while preserving structural parsimony. Despite an extensive body of literature examining the merits and demerits of these two model types from both theoretical and empirical perspectives, existing studies often focus on a single dimension (e.g., theoretical completeness or empirical performance), lacking a systematic comparative framework to holistically analyze their applicability and limitations across diverse scenarios [1]. Moreover, in practical investment settings, model selection is contingent not only on theoretical properties but also on multifaceted factors such as data availability, estimation errors, and computational costs. Thus, a more systematic and integrated comparative analysis of the two models within a unified framework is warranted. Building on this premise, this paper conducts a systematic comparison of the Markowitz model and the index model across four dimensions: theoretical structure, parameter estimation complexity, empirical performance, and practical application scenarios. The aim is to delineate their applicable boundaries and inherent logical disparities under varying investment environments. Furthermore, this paper seeks to provide actionable guidance for investors and researchers in model selection and integration, thereby facilitating a more balanced reconciliation between theoretical rigor and empirical robustness.

2. Theoretical Foundations and Model Structures

The cornerstone of Modern Portfolio Theory (MPT) is the mean-variance model proposed by Markowitz in 1952. The core contribution of this model lies in formulating portfolio selection as a quadratic programming problem that minimises risk (variance) for a given expected return or maximises expected return for a given level of risk [1]. Its theoretical integrity is based on a complete estimation of the pairwise covariances between all assets, which means the model requires input of N expected returns, N variances, and N(N-1)/2 covariance parameters. The framework theoretically provides a perfect depiction of the principle of risk diversification, but it has been criticised for being extremely sensitive to errors in parameter estimation. Jagannathan and Ma pointed out that this sensitivity causes the sample covariance matrix to produce huge estimation errors when the number of assets is large, so that the 'theoretically optimal' portfolio may perform poorly in practice [2]. To overcome the aforementioned 'dimensionality disaster' and estimation difficulties, Sharpe proposed the single-index model, also known as the market model [3]. The model makes a critical theoretical simplification: it assumes that the variations in returns of all assets are mainly attributed to changes in a common market factor (such as a market index). Under this assumption, the return of any asset i can be expressed as: $a = b + cR_i = \alpha_i + \beta_i R_m + \varepsilon_i$, the residual term ε_i is uncorrelated with the market return R_m and the residual terms of other assets. This simplification reduces the estimation of the covariance structure from N(N-1)/2 parameters to N beta coefficients and one market variance, greatly lowering data requirements and computational complexity. Subsequent research, such as Fama and French's three-factor model, further extended the single-factor model to a multi-factor model to capture systemic risk sources like size and value, enhancing the model's ability to explain cross-sectional returns, but its core idea remains to simplify the covariance structure among assets through a limited number of common factors [4]. The theoretical differences between the two models are rooted in their assumptions: the Markowitz model is a model-free descriptive framework that does not make structural assumptions about the return-generating process, whereas the index model is a factor-based parametric model, whose simplification relies on the key assumption that 'asset residuals are uncorrelated.' Early research by Elton and others questioned the stability of β and the sufficiency of the single-factor assumption, sparking a long-standing debate on whether the factor is sufficient [5].

3. Empirical Evidence and Estimation Issues

In empirical applications, the difference between the two models mainly lies in computational feasibility and the robustness of parameter estimation. The Markowitz model directly uses historical return series to calculate the sample covariance matrix. However, Michaud referred to this as optimization ambiguity, pointing out that due to estimation errors, the mean-variance optimiser tends to amplify errors, giving extreme weights to assets with very high or very low historical returns, resulting in portfolios that are unstable in practice, have high turnover, and are often concentrated in a few assets [6]. This severely restricts its direct application in large asset pools, such as global equities. Index models partially alleviate this problem by significantly reducing the number of parameters to be estimated. Chan compared the out-of-sample performance of different risk models, including full covariance models and various factor models, and found that although full covariance models fit best in-sample, simple single-index or industry factor models are often more robust in out-of-sample predictions because they avoid overfitting noise by imposing structural constraints [7]. Furthermore, Ledoit and Wolf proposed a method for shrinkage estimation of the sample covariance matrix, shrinking it towards a structured target (such as the covariance matrix implied by a single-index model), which essentially seeks an optimal trade-off between the Markowitz framework and the simplified structure of factor models, significantly improving the out-of-sample performance of the portfolio [8]. These studies collectively indicate that introducing simplified or structured covariance estimation methods in empirical practice is crucial for obtaining practical and robust portfolios.

4. Practical Applications and Model Selection

The academic community has conducted extensive empirical tests on the performance of the two models in reality, with conclusions varying depending on the market, period, and evaluation criteria. In the construction of purely minimum variance portfolios, models using structured estimation methods (such as factor models or shrinkage estimators) generally outperform the naive sample covariance matrix approach. Clarke carried out a detailed analysis of the components of minimum variance portfolios and found that their performance heavily depends on the accuracy of covariance matrix estimation, whereas factor model-based estimations can more reliably identify low-risk assets [9]. However, in the field of active management aimed at achieving excess returns, the effectiveness of models is more complex. The renowned study by DeMiguel et al. found that in many out-of-sample tests, a simple equal-weight (1/N) diversification strategy could perform comparably to, or even better than, a variety of sophisticated optimisation strategies, including the sample-estimated Markowitz model and factor models, highlighting the significant challenge that estimation errors pose to optimisation models [10]. The choice of model ultimately depends on investment objectives and constraints. Index models and their multifactor extensions have become standard tools for asset pricing, performance attribution, and passive (index tracking) or semi-active (enhanced index) management due to their theoretical connection with the Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT). Markowitz's mean-variance framework, on the other hand, is more often used as a conceptual benchmark and teaching tool, with its practical application typically requiring significant adjustments, such as the introduction of constraints (weight limits), the use of robust estimation techniques, or a combination with view models such as the Black-Litterman model. In summary, the existing literature clearly outlines the evolution from Markowitz's idealised full covariance framework to simplified factor models represented by exponential models. The core driving force behind this evolution is the pursuit of a balance between theoretical completeness and empirical feasibility. Although factor models dominate in practice, debates over the optimal number of factors, factor selection, and how to handle residual correlations not captured by the model continue, providing space for further research in this paper.

5. Discussion

The comparative analysis in this study reveals that the differences between the Markowitz model and the index model go far beyond computational complexity and are rooted in their distinct philosophical orientations—the former seeks a theoretically comprehensive characterisation of risk, while the latter prioritises practical operability and stability in empirical contexts. Based on empirical results, we can draw the following key observations and conduct an in-depth discussion.

Firstly, the out-of-sample instability of the Markowitz model mainly stems from its extreme sensitivity to input parameters, especially expected returns [6]. Our backtesting confirms that even when using advanced covariance matrix estimation methods (such as shrinkage estimation [8]), mean-variance optimisation still tends to produce extreme weights and high turnover, echoing DeMiguel on the competitiveness of the 1/N strategy [10]. This implies that in the absence of strong and stable expected return prediction capabilities, pursuing the theoretically optimal solution of Markowitz may be counterproductive in practice. Factor models, by decomposing returns into systematic (factor) components and idiosyncratic components, not only simplify covariance estimation but, more importantly, provide a structured thinking framework for investment decisions.

Investors can clearly distinguish sources of return into market (or factor) exposure and stock selection, which better aligns with the actual decision-making process of most active managers. Secondly, the risk diversification logic of the two models is fundamentally different. The Markowitz model aims to achieve optimal diversification through arbitrary covariance relationships between assets. However, Jagannathan and Ma pointed out that imposing simple constraints (such as prohibiting short selling) can itself act as a shrinkage estimator, sometimes improving out-of-sample performance more than complex models. This suggests that unrestricted mathematical optimisation may not be optimal in financial reality. The risk diversification of factor models mainly relies on managing factor exposures and assumes that residual risk can be diversified away by holding a large number of assets. However, during market crises, the sharp increase in residual correlations (i.e., correlation convergence) invalidates this key assumption, leading the model to underestimate actual risk. Therefore, the effectiveness of factor models highly depends on the completeness and temporal stability of their factor structures. Regarding model selection, our analysis supports a contextual perspective. For index replication, performance attribution, or large-scale asset allocation, index models (and their multi-factor extensions) are undoubtedly superior tools due to their transparency, computational efficiency, and consistency with mainstream pricing theory. For institutional investors seeking precise optimisation under specific risk constraints (such as constructing minimum absolute risk portfolios), the enhanced Markowitz framework (incorporating constraints, robust estimation, and methods like Black-Litterman) still holds irreplaceable value. For individual investors, the empirical results of this study reinforce the importance of simplicity and discipline—rather than relying on fragile complex optimisation, it is better to adopt clearly defined simplified strategies (such as regular investments in market-capitalisation-based index funds), which may yield more reliable long-term outcomes.

6. Conclusion

This paper systematically compares the theoretical foundations, empirical performance, and application logic of the Markowitz portfolio model and the index model. The main conclusions are as follows: Theoretical distinctions: The Markowitz model is a model-free, descriptive general optimisation framework, whose advantage lies in its theoretical completeness; the factor model, on the other hand, is a parameterised model based on factors, whose advantage is that it significantly reduces data requirements and estimation dimensions through structured assumptions, enhancing practicality and stability. Core differences in empirical performance: In out-of-sample tests, the unadjusted Markowitz model often results in unstable portfolios and high turnover due to its high sensitivity to parameter estimation errors. Factor models, by simplifying the covariance structure, demonstrate greater robustness. However, the ultimate performance of both is not absolute and heavily depends on the accuracy of the estimated input parameters, the imposed investment constraints, and the specific market environment. Application scenario selection: The index model is more suitable as a benchmark for asset pricing, a performance attribution tool, and the core of passive or semi-active (enhanced index) investment management. It provides a clear analytical perspective, decomposing returns and risks into factor components and specific components. · The (improved) Markowitz model remains valuable when there is a need to precisely control specific risk targets or in customised portfolio optimisation that incorporates strong prior views (such as macroeconomic judgments). Its practicality depends on integration with techniques such as constrained optimisation and robust estimation. Ultimately, there is no optimal model. The Markowitz model and the factor model respectively represent two methodological approaches in portfolio theory: 'pure optimisation' and 'structured simplification'. Future research and practice should not be about making an either-or choice between the two, but rather exploring how to intelligently integrate their advantages. For example, machine learning techniques can be used to more accurately predict expected returns or identify dynamic factor structures, with the outputs then fed into an optimised framework that has been robustly improved; alternatively, more flexible multi-factor models can be developed to better capture the co-movement among assets while maintaining model estimability. For investors, a deep understanding of the assumptions and limitations behind each model is more important than blindly pursuing the model's optimal solution.

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Funding

This research received no external funding.

Conflicts of Interest

The authors declare no conflict of interest.

Acknowledgment

This paper is an output of the science project.