

From Motion Explanation to Control Guidance: A Review of the Inverted Pendulum, Compass Gait, and SLIP in Bio-Inspired Legged Robotics

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Abstract

Minimal models have played a central role in legged locomotion research by reducing complex biological and robotic systems to interpretable dynamical templates. This review examines three representative models—the inverted pendulum, compass gait, and spring-loaded inverted pendulum (SLIP)—to clarify their theoretical significance and continuing relevance to bio-inspired legged robotics. Through a comparative review of representative studies, the paper analyses how these models explain locomotion mechanics, support gait analysis, and inform robot control and design. The analysis shows that the inverted pendulum captures the global mechanics of walking and centre-of-mass motion, the compass gait highlights passive dynamics, hybrid transitions, and gait stability, and the SLIP model emphasises compliance, elastic energy storage, and dynamic efficiency. Taken together, the three models form a progressive theoretical chain from motion explanation to control-oriented abstraction. Although their simplifying assumptions limit direct application to modern robotic platforms, they remain valuable as interpretable and transferable templates for analysing locomotion, structuring controller design, and guiding future model extension. This review therefore argues that classical minimal models continue to provide an important conceptual bridge between biomechanics, dynamical analysis, and robotic implementation.

Keywords

bio-inspired robotics, legged locomotion, minimal models, compass gait, robot control

1. Introduction

Legged locomotion remains a central topic in bio-inspired robotics because it enables mobility in environments where wheeled or tracked systems are limited by uneven terrain, obstacles, and discontinuous ground contact [1, 2]. At the same time, biological and robotic legged systems are dynamically complex, involving multi-body motion, intermittent contact, compliance, and feedback control [2, 3]. For this reason, researchers have long relied on minimal models to extract the dominant principles of locomotion from otherwise high-dimensional systems. Their value lies not only in simplification, but also in providing low-dimensional and interpretable templates for analysing centre-of-mass motion, energetic exchange, gait stability, and control structure [1, 2].

Among these models, the inverted pendulum, compass gait, and spring-loaded inverted pendulum (SLIP) have become three of the most influential theoretical templates for legged locomotion [1, 2]. The inverted pendulum explains walking as a vaulting motion over a stance leg and captures the exchange between gravitational potential and kinetic energy during walking [4, 5]. The compass gait extends this perspective by introducing alternating stance and swing legs, enabling the analysis of passive dynamic walking, hybrid impact dynamics, and limit-cycle stability [6-8]. The SLIP model, by contrast, emphasises leg compliance and elastic energy storage, and has therefore become a canonical framework for explaining running, hopping, and compliant walking [9-11].

Despite their influence, these models are often discussed separately, with more emphasis on their individual dynamics than on their broader conceptual relationship [2, 3]. Yet contemporary legged robots still depend on simplified templates even as they incorporate compliance, sensing, whole-body coordination, and advanced control [12, 13]. A comparative synthesis is therefore needed to assess both the enduring value and the practical limits of classical minimal models in modern legged robotics.

Against this background, this review addresses three questions: what locomotion principles are captured by the inverted pendulum, compass gait, and SLIP models; how have these models contributed to robot analysis, control, and design; and what are their principal limitations and continuing values in contemporary legged robotics? Based on these questions, this review is guided by three research objectives. First, it aims to identify the key locomotion principles captured by the inverted pendulum, compass gait, and SLIP models. Second, it compares how these models have contributed to gait analysis, stability interpretation, robot control, and mechanical design. Third, it evaluates the limitations of these classical minimal models and discusses their continuing value for contemporary bio-inspired legged robotics.

To address these questions and objectives, the paper compares representative studies across biomechanics and robotics and is organised around three model-based sections: the inverted pendulum, compass gait, and SLIP. This structure allows the review to clarify the theoretical significance, engineering relevance, and application boundaries of minimal models within the broader development of bio-inspired legged robots.

2. Literature Review

2.1 Inverted Pendulum Model and the Explanation of Walking

The inverted pendulum model is one of the earliest and most influential simplified models in legged locomotion research. In its basic form, the body is represented as a concentrated mass supported by a relatively rigid stance leg, so that the centre of mass follows an arc-like trajectory as the body vaults over the supporting limb [4, 5]. This abstraction captures a key feature of walking: during stance, gravitational potential energy and kinetic energy are exchanged in a partially out-of-phase manner, reducing the mechanical work required for forward progression [4, 5]. For this reason, the inverted pendulum has long served as a fundamental framework for explaining the mechanics of walking in humans and animals.

A major contribution of the model is that it clarifies why walking differs mechanically from running. In walking, the centre of mass reaches its highest position near mid-stance, consistent with pendular motion, whereas running depends more strongly on compliance and elastic energy storage [4, 5]. The model therefore provides a simple explanation for the characteristic centre-of-mass trajectory of walking and for the energetic logic that makes walking economical at moderate speeds [4, 5, 14]. It has also been used to interpret preferred speed-step relationships and gait selection [14, 15].

In bio-inspired robotics, the importance of the inverted pendulum extends beyond biological explanation. Although highly simplified, it provides a low-dimensional template for analysing balance, forward progression, and centre-of-mass behaviour without immediately confronting the full complexity of multi-joint robot dynamics [1, 2, 15]. At the same time, it cannot represent swing-leg dynamics, impact events, leg compliance, or the distributed control requirements of real robotic systems [2, 15]. Its significance, therefore, lies less in completeness than in establishing a foundational language for reasoning about walking as a dynamical process and a starting point for later minimal models [1, 2].

2.2 Compass Gait and Passive Dynamic Walking

The compass gait model extends the logic of the inverted pendulum by introducing a two-link leg structure and alternating stance and swing phases, thereby providing a more explicit representation of periodic bipedal walking [2, 6]. In its classical form, it consists of two rigid legs connected at the hip, typically without knees and with little or no actuation [6, 7]. Despite this simplicity, the model captures a feature that the inverted pendulum cannot fully represent: walking is not only centre-of-mass vaulting, but also a hybrid dynamical process involving continuous motion and discrete leg-exchange events [2, 7, 8].

A major contribution of the compass gait model is that it demonstrates how walking-like behaviour can emerge from the natural dynamics of a mechanical system rather than from continuous high-level control alone. McGeer's work on passive dynamic walking showed that, under suitable conditions such as a slight slope and appropriate morphology, a simple biped can generate stable and repeatable gait patterns with little or no active actuation [6]. Later studies further analysed these behaviours in terms of symmetry, periodicity, and stability, showing that passive walking can arise from the interaction between gravity, body geometry, and impact dynamics [7, 8]. This challenged the assumption that stable walking necessarily requires heavy actuation or strict trajectory tracking.

The model also introduced a clearer treatment of hybrid gait dynamics. By explicitly including continuous evolution and discrete foot-contact transitions, it became useful for analysing step-to-step transitions, periodic orbits, and gait stability [7, 8, 16]. In robotics, this insight strongly influenced the design of passive-dynamic and underactuated walkers, including experimental robots that achieved remarkably energy-efficient bipedal locomotion by exploiting natural mechanics [17, 18]. Although the model remains highly idealised and cannot capture compliant contact, whole-body coordination, or modern sensing and actuation demands [2, 16], it provided a crucial bridge between simple walking mechanics and later control-oriented models of legged locomotion.

2.3 SLIP and the Spring-Leg Paradigm

The spring-loaded inverted pendulum (SLIP) model represents a further development in minimal locomotion modelling by introducing leg compliance as a central dynamic feature. In its standard form, the body is modelled as a point mass supported by a massless spring-like leg during stance, while flight or leg-exchange phases connect successive steps [9, 19]. Unlike the rigid-leg logic of the inverted pendulum and compass gait models, the SLIP framework captures the ability of biological and robotic legs to store, release, and regulate mechanical energy through compliant interaction with the ground [2, 9]. For this reason, it has become one of the most influential templates for explaining running, hopping, and, in extended forms, compliant walking [9, 10].

A major contribution of the SLIP model is that it explains locomotion as a spring-mass process rather than as rigid-body vaulting or passive leg exchange. In running and hopping, the centre of mass descends during stance as elastic elements deform and rises again as stored energy is returned. The model captures this behaviour in a compact form, showing how compliant legs can support dynamic stability, attenuate impact, and improve locomotor efficiency [9, 19]. Later work showed that compliant leg behaviour can also account for some features shared by walking and running, extending the model's relevance beyond running alone [10].

In bio-inspired robotics, the SLIP model is especially important because it is closely connected to robots with elastic actuation, compliant structures, or dynamic running capability [1, 2]. It provides a basis for analysing how leg stiffness, touchdown angle, body trajectory, and energy regulation affect locomotor performance [19, 20], and it has informed both the control of hopping and running robots and broader design philosophies that emphasise embodied dynamics [11, 19, 20]. Although it remains an abstraction that cannot fully represent multi-joint structure, sensing uncertainty, or detailed contact mechanics [2, 11], it introduced compliance as a central organising principle in minimal locomotion theory and provided an important bridge between biomechanics and robot control [9-11].

3. Key Findings

Taken together, the inverted pendulum, compass gait, and spring-loaded inverted pendulum (SLIP) should be understood not as isolated models, but as a progressive theoretical chain in the development of bio-inspired

legged robotics [1, 2]. The inverted pendulum provides the simplest abstraction of walking through centre-of-mass vaulting and pendular energy exchange [4, 5]. The compass gait extends this logic by introducing alternating stance and swing phases, thereby shifting minimal modelling toward hybrid, step-to-step gait dynamics [6-8]. The SLIP model moves further by treating compliance and elastic energy storage as central principles of locomotion [9, 10]. These models therefore represent not only separate abstractions, but also successive expansions in what minimal models can explain.

Their modelling assumptions reflect a corresponding development in theoretical scope. The inverted pendulum mainly captures the global mechanics of walking [4, 5, 14]. The compass gait incorporates leg exchange and impact transitions, making it more suitable for analysing passive dynamic walking and hybrid stability [6-8, 16]. The SLIP model departs from rigid support by introducing spring-like leg behaviour, allowing it to account more directly for impact attenuation, energy return, and dynamically efficient locomotion [9, 10, 19]. In this sense, the three models reflect different analytical priorities: overall body motion, hybrid gait structure, and compliant legged movement.

The comparison also reveals a broader transition from motion explanation to control guidance. The inverted pendulum is most closely associated with explanatory biomechanics [4, 5, 14]. The compass gait marks a clearer turn toward control-oriented reasoning by showing how stable gait can emerge through morphology, gravity, and limited actuation [6-8]. The SLIP model extends this trend by providing a dynamic template directly relevant to the design and control of compliant robots [11, 19, 20]. Their value therefore lies not in fully representing real robots, but in isolating dominant dynamic principles while remaining relevant to robot analysis and design [1-3].

The findings of this review suggest that the three models should be treated as complementary theoretical templates rather than competing alternatives [1, 2]. The inverted pendulum explains basic walking mechanics and centre-of-mass behaviour [15], the compass gait provides a clearer account of passive dynamic stepping and hybrid transitions [16], and the SLIP model extends minimal modelling toward compliant and energy-efficient locomotion [11]. Their broader significance lies in showing how minimal modelling in legged robotics evolved from motion explanation to control-oriented abstraction and mechanical design [1-3].

4. Discussion

The findings above also reveal the application boundaries of classical minimal models. All three rely on strong simplifying assumptions, including rigid or massless limbs, reduced body representation, idealised contact events, and restricted control conditions [2, 3]. These assumptions are valuable for theoretical clarity, but they limit direct transfer to modern robotic systems, which must coordinate multiple joints, cope with actuator constraints, process uncertain sensory information, adapt to irregular terrain, and satisfy task-specific objectives [3, 12, 13]. Minimal models are therefore best understood as theoretical anchors rather than complete control frameworks. Their practical value depends on how effectively their principles can be embedded within richer modelling and control architectures [3, 13].

Their continued relevance also depends on interpretability. Contemporary legged robotics increasingly uses whole-body control, optimisation, model predictive control, and learning-based methods, but these do not remove the need for simple physical templates [12, 13, 21, 22]. On the contrary, interpretable models remain valuable because they help explain why a gait works, which variables dominate the motion, and where stability or efficiency originates [1-3]. In this sense, minimal models can provide physical structure for modern control methods rather than competing with them.

Future research should therefore focus on strengthening the connection between classical minimal models and practical robotic implementation. One important direction is to embed simple templates such as the inverted pendulum, compass gait, and SLIP within richer whole-body models, so that their interpretability can be preserved while more realistic features such as joint actuation, distributed mass, foot-ground interaction, and actuator limits are included [3, 12, 13]. A second direction is to develop adaptive control frameworks in which the parameters of minimal models, such as leg stiffness, touchdown angle, or centre-of-mass trajectory, can be adjusted online according to terrain conditions and task requirements [11, 19-21].

A further direction is to combine template-based modelling with optimisation-based and learning-based methods. In this role, minimal models would not replace modern control algorithms, but would provide

physical structure, reduce the search space, and improve the interpretability of learned or optimised behaviours [1, 2, 13, 21, 22]. Future bio-inspired legged robotics is therefore likely to benefit most from a hybrid approach, where classical minimal models provide the conceptual and dynamical foundation, while advanced sensing, computation, and control methods provide the adaptability required for real-world operation [3, 12, 13, 22].

In addition, future work should pay closer attention to experimental validation of template-based principles on physical robots. The classical models reviewed here are often evaluated through reduced-order dynamics or idealised assumptions, but their practical relevance depends on whether the same principles remain observable under actuator saturation, sensor noise, impact losses, and uncertain ground contact. For this reason, future studies should compare template-level predictions with full multibody simulation and hardware experiments across different terrains and speeds. Such validation would help identify which variables can be safely abstracted and which practical constraints must be retained when minimal models are used for controller design. This would further strengthen the role of minimal models as interpretable bridges between theoretical locomotion principles and deployable robotic systems [3, 12, 13, 21, 22].

5. Conclusion

This review has examined three representative minimal models in bio-inspired legged robotics: the inverted pendulum, compass gait, and spring-loaded inverted pendulum (SLIP). The analysis has shown that these models capture different but connected locomotion principles. The inverted pendulum explains the basic mechanics of walking and centre-of-mass motion, the compass gait reveals the importance of passive dynamics and hybrid gait stability, and the SLIP model highlights compliance, elastic energy storage, and dynamic efficiency in legged locomotion. Taken together, they form a clear theoretical progression from motion explanation to increasingly control-relevant abstractions of gait generation and locomotor performance.

The significance of these models lies not in fully representing real biological or robotic systems, but in providing interpretable and transferable templates for analysis. Their lasting contribution to legged robotics is that they reduce complex locomotion to dominant dynamic principles that support gait interpretation, stability reasoning, and control-oriented design. At the same time, their simplifying assumptions limit direct use in modern robotic systems. They should therefore be understood not as complete solutions, but as foundational abstractions whose value depends on how effectively they are extended and integrated into richer robotic frameworks.

Overall, minimal models remain important in contemporary legged robotics because they preserve conceptual clarity in a field of increasing technical complexity. Even as legged robots incorporate more advanced hardware, optimisation-based methods, and learning-based control, classical minimal models continue to provide an essential basis for linking locomotion theory with robot design, controller development, and future model extension.

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Conflicts of Interest

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