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From Black-Scholes to Jump-Diffusion: A Narrative Review of Option Pricing Model Evolution

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Abstract

Option pricing theory is a cornerstone of modern financial engineering, but the classical Black-Scholes framework relies on continuous price trajectories, constant volatility, and lognormally distributed returns. These assumptions are difficult to reconcile with empirical features such as volatility smiles, fat-tailed returns, volatility clustering, and abrupt price movements. This review examines the evolution of Option pricing models from the Black-Scholes benchmark to stochastic-volatility, jump-diffusion, stochastic-volatility-jump, and affine jump-diffusion specifications. It shows that richer models generally improve cross-sectional pricing accuracy, especially for short-maturity and out-of-the-money options, but that this improvement is accompanied by higher parameter uncertainty, calibration complexity, and hedging instability. The review further emphasizes that evidence from emerging markets, particularly China's domestic derivatives market, remains comparatively limited and may require models that account for liquidity constraints, retail participation, and clustered jump behavior.

Keywords

option pricing, jump-diffusion models, stochastic volatility, affine processes, emerging market options

1. Introduction

Option pricing theory transformed modern finance by providing a rigorous mechanism for valuing derivative securities and managing nonlinear risk. The Black-Scholes (B-S) model [1], together with Merton's rational Option pricing framework [2], established the no-arbitrage benchmark under continuous trading, constant volatility, and lognormally distributed returns. However, market evidence soon showed that these assumptions were too restrictive. Empirical studies found systematic pricing errors and implied-volatility patterns inconsistent with constant volatility, particularly for out-of-the-money options and after major market stress episodes [3,4].

The purpose of this review is therefore not to restate the standard chronology of Option pricing models, but to synthesize why each structural extension emerged and where it remains limited. Existing reviews and empirical summaries have tended to focus on mature U.S. and European option markets, while the institutional and microstructure features of emerging markets have received less integrated attention. This gap matters

because jump clustering, liquidity frictions, and retail-driven trading may affect option prices differently from the patterns observed in highly liquid index-option markets. Accordingly, this paper connects the main theoretical line from jump-diffusion [5], stochastic volatility [6], stochastic-volatility-jump models [7], and affine jump-diffusion frameworks [8] with transform-based valuation methods [9] and recent evidence from China's option markets [10].

This review addresses three research questions: (1) how have Option pricing models evolved structurally in response to the limitations of the B-S model? (2) What empirical evidence supports the superiority of jump-diffusion or stochastic-volatility-jump models over simpler alternatives? (3) What practical gaps remain when these models are applied to emerging markets, particularly China?

The research value of this paper lies in linking model design to market applicability. By comparing pricing accuracy, hedging stability, parameter risk, and emerging-market suitability within one review, the paper clarifies why more complex models are not automatically better in practice and identifies concrete directions for China-based empirical testing.

2. Theoretical Background

2.1 Foundational Concepts and the B-S PDE

At the center of option pricing is the modeling of the underlying asset's price dynamics. The B-S model assumes that the asset price S_t follows a geometric Brownian motion (GBM):

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t \tag{1}$$

where μ denotes the drift, σ is the constant volatility, and W_t is a standard Wiener process. Under the risk-neutral measure, this assumption leads to the Black-Scholes partial differential equation (PDE) for the price $C(S,t)$ of a European call option:

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC \tag{2}$$

where r denotes the risk-free interest rate. Despite its theoretical importance, the B-S framework restricts the underlying asset to a smooth and continuous price path with constant variance, which is inconsistent with several well-documented market features.

2.2 Beyond Black-Scholes: Jumps and Stochastic Volatility

To bring theory closer to observed market behavior, later models relaxed the B-S assumptions along two main dimensions: discontinuous price jumps and time-varying volatility. These two extensions address different empirical failures. Jump models focus on rare but large price movements and hence are directly related to tail risk. Stochastic-volatility models focus on persistent changes in conditional variance and hence are better suited to volatility clustering. Neither extension is complete on its own, which explains why later research moved toward integrated specifications.

Jump-diffusion models. Merton [5] augmented the diffusion process with a Poisson-driven jump component to capture discontinuous price changes caused by earnings surprises, macroeconomic announcements, or geopolitical shocks. The dynamics can be expressed as follows:

$$dS_t = \mu S_t dt + \sigma S_t dW_t + S_{t^-} dJ_t \tag{3}$$

where the jump component is governed by a Poisson process with intensity, and the log-jump size is commonly assumed to follow a normal distribution. The advantage of this specification is its direct treatment of tail events; its limitation is that standard normally distributed jumps may not fully capture asymmetry or clustered jump arrivals. Later double-exponential jump specifications partially address this asymmetry by allowing different probabilities for upward and downward jumps [11].

Stochastic volatility. Heston [6] addressed the constant-volatility assumption by modeling variance as a mean-reverting square-root process, building on the broader stochastic-volatility approach to option valuation [12]. The process can be written as follows:

$$dv_t = \kappa(\theta - v_t)dt + \sigma_v\sqrt{v_t}\,dW_t^v \tag{4}$$

where mean reversion, long-run variance, and volatility of volatility determine the evolution of conditional variance. The strength of stochastic volatility is that it generates smiles and term-structure effects without imposing abrupt price discontinuities. Its weakness is that it may understate crash risk when sudden jumps are an important source of market movement.

2.3 Limitations of the B-S Framework

The need for these extensions becomes evident when B-S assumptions are compared with empirical market data. First, realized returns display leptokurtosis, meaning that the empirical distribution has fatter tails and a sharper peak than the normal distribution implied by the B-S model. Second, implied volatility is not constant across strike prices; instead, it often forms a U-shaped volatility smile or a downward-sloping volatility smirk. Deep in-the-money and out-of-the-money options therefore command higher implied volatilities than at-the-money options. Third, the same model can perform differently across markets: a specification calibrated to a liquid U.S. index-option market may be less reliable in an emerging market where trading activity is thinner, investor composition differs, and jumps may occur in clusters. These deviations show that model choice should be judged not only by theoretical elegance but also by empirical fit, parameter stability, and market setting.

2.4 The Affine Jump-Diffusion (AJD) Unification

Duffie, Pan, and Singleton [8] unified the above ideas through the affine jump-diffusion framework. The core innovation is that state variables such as volatility, interest rates, or jump intensity can be specified in an affine form, so the conditional characteristic function of the asset price remains exponentially affine in those state variables. This structure makes it possible to combine stochastic volatility, jumps, and sometimes stochastic interest rates in one internally consistent model without losing analytical tractability.

The importance of the AJD framework lies in its balance between flexibility and computability. Fourier inversion and related transform methods can recover option prices from the characteristic function much faster than repeated simulation, while still allowing the model to fit volatility smiles and term structures [9]. Subsequent cosine-expansion methods further improved numerical efficiency for European-style options [13]. This is why AJD-type models became a mainstream framework in derivatives research: they provide a practical bridge between rich economic dynamics and feasible calibration.

3. Literature Review

The literature can be organized into four stages, but the main issue is not chronology alone. Each stage reflects a specific trade-off between empirical realism, analytical tractability, and implementation risk. The following review therefore combines historical development with evaluative comments on what each stage solved and what it left unresolved.

3.1 The Foundational Era (1973–1980S)

The decade following the B-S model focused on establishing the no-arbitrage pricing paradigm. Black and Scholes [1] and Merton [2] provided the theoretical foundation under continuous trading and constant volatility. Although this framework was revolutionary, empirical tests by Rubinstein [3] and Jackwerth and Rubinstein [4] showed that the B-S model systematically mispriced out-of-the-money options. These errors were closely related to implied-volatility smirks, which suggested that the market was pricing crash risk and time-varying volatility even when the basic model did not allow for them.

This foundational era is best understood as a benchmark-building stage rather than a complete empirical explanation. Its main contribution was to define the no-arbitrage logic and replicating-portfolio method that later models retained. Its main weakness was that the assumptions required for a closed-form solution also removed the very features--fat tails, skewness, and changing volatility--that are central to real option prices.

3.2 Single-Factor Extensions (1980S–1990S)

Researchers first addressed B-S failures through single-factor extensions. Merton [5] introduced a jump-diffusion model and derived a tractable solution for European options with discontinuous price movements. Kou [11] later improved the jump specification by allowing asymmetric double-exponential jumps, which better reflect the different market responses to upward and downward shocks. In parallel, Heston [6] developed a stochastic-volatility model that relaxed the constant-volatility assumption and used characteristic functions to obtain a semi-analytical solution. Hull and White [12] also showed that stochastic volatility can materially affect option values when volatility risk is not constant.

The contribution of this stage was to identify two separate sources of B-S misspecification: discontinuous returns and random variance. However, single-factor extensions remained incomplete. A pure jump-diffusion model can capture crash risk but often treats volatility too rigidly, whereas a pure stochastic-volatility model can capture volatility clustering but may miss sudden discontinuities. This complementarity created the theoretical motivation for multi-factor integration.

3.3 Multi-Factor Integration and Analytical Tractability (1990S–2000S)

As neither jumps nor stochastic volatility alone fully explained market dynamics, researchers began to integrate both features. Bates [7] combined Heston's stochastic-volatility framework with Merton-type jumps to form an SVJ model and showed that this hybrid specification explained exchange-rate option smiles more effectively than single-factor alternatives. Computational barriers were reduced when Duffie, Pan, and Singleton [8] formalized the AJD framework, which allowed researchers to apply Fourier-transform methods [9] and cosine expansions [13]. These techniques substantially reduced the computational burden associated with option valuation, especially relative to simulation-intensive methods.

The key contribution of multi-factor integration was synthesis: it recognized that tail risk and volatility clustering are not competing explanations but interacting channels. The limitation is that every added factor increases calibration difficulty. SVJ and AJD models may fit option surfaces better, but they also require more parameters and stronger assumptions about jump intensity, jump-size distribution, and volatility dynamics. This makes their out-of-sample and hedging performance less certain than their in-sample pricing performance.

3.4 Empirical Validation and High-Frequency Advancements (2000S–Present)

Once tractable models became available, empirical validation expanded rapidly. Bakshi, Cao, and Chen [14] and Broadie, Chernov, and Johannes [15] found that models incorporating stochastic volatility and jumps can substantially reduce pricing errors relative to simpler diffusion models, especially for options away from the money. Eraker [16] further showed that jumps are important for reconciling evidence from spot and option prices. More recent research has shifted attention toward model features that standard AJD specifications may not fully capture. Pan and Ma [10] report self-exciting jump behavior in China's CSI 300 ETF options, while rough-volatility research argues that volatility can exhibit rough, persistent behavior at high frequency [17].

This recent literature shows that the standard AJD framework is powerful but not final. Its common Poisson-jump structure may understate jump clustering, and its Markovian volatility structure may miss rough volatility patterns. Hawkes-type jumps and rough-volatility extensions therefore represent more than technical refinements; they respond to observable features of modern option markets, particularly markets where liquidity shocks and investor herding can make jump arrivals self-reinforcing.

4. Discussion and Synthesis

4.1 Consistencies and Empirical Dominance

According to Table 1, a consistent finding is that the B-S model is misspecified for modern liquid markets because it cannot adequately capture tail risk or dynamic volatility. The reason more complex models improve pricing is structural rather than mechanical: stochastic volatility changes the conditional variance path, jumps add discontinuous tail events, and integrated SVJ or AJD models allow both channels to affect option values simultaneously. This is why SVJ-type models are particularly useful for short-maturity and out-of-the-money options, where crash risk and volatility skew are most visible [14-16].

Table 1: Comparative synthesis of major Option pricing model classes

Model	Pricing accuracy	Hedging performance	Suitable settings	Main limitations
B-S	Low when smiles, smirks, and fat tails are strong.	Simple and stable Greeks under ideal assumptions.	Benchmark pricing, teaching, quick at-the-money estimates.	Constant volatility, continuous paths, lognormal returns.
SV	Improves smile and term-structure fit through random variance.	Often more stable than jump-rich models but sensitive to volatility risk.	Markets dominated by volatility clustering rather than abrupt jumps.	Cannot fully capture crash jumps or discontinuous price moves.
SVJ	Strong for short-maturity and out-of-the-money options.	More difficult because jump risk is hard to hedge dynamically.	Markets with news shocks, crash risk, or pronounced skew.	More parameters; jump premia and intensities can be unstable.
AJD	High flexibility with tractable transform-based valuation.	Depends on stable state-variable and jump-parameter calibration.	Institutional calibration across volatility surfaces and maturities.	Standard affine assumptions may miss rough volatility and self-exciting jumps.

At the same time, model superiority is conditional. A model that fits the implied-volatility surface more closely may not be superior for every purpose. For pricing, richer state dynamics help explain curvature and skew. For hedging, however, additional jump and volatility-risk factors create exposures that cannot always be replicated continuously. The empirical dominance of SVJ and AJD models should therefore be interpreted as strongest for pricing and calibration, not as a general guarantee of hedging superiority.

4.2 Contradictions, Trade-Offs, and Limitations

The main contradiction in the literature is the trade-off between pricing accuracy and implementation reliability. Complex models often reduce in-sample pricing errors, but they require estimates of volatility mean reversion, volatility of volatility, jump intensity, jump-size distribution, risk premia, and sometimes stochastic interest-rate parameters. These estimates can be unstable when option data are noisy, sparse, or concentrated in a small set of maturities. Model uncertainty can therefore become economically important even when the model is mathematically coherent [18].

The mature-market versus emerging-market distinction is central to this trade-off. In mature U.S. and European index-option markets, deeper liquidity and broader institutional participation make continuous calibration more feasible. In emerging markets, including China, option trading may be more affected by retail behavior, policy news, margin constraints, and intermittent liquidity. These features can produce clustered jumps and discontinuous changes in implied volatility that standard Gaussian or Poisson assumptions do not fully capture [10]. As a result, the most suitable model for China may not simply be the most complex imported model, but a specification that balances jump clustering, rough volatility, data availability, and recalibration stability.

4.3 Future Research Directions

The limitations identified above point to three China-relevant research trajectories that can be developed as complete empirical programs rather than as isolated technical extensions.

First, high-frequency calibration should be used to test whether intraday volatility bursts and jump arrivals in Chinese ETF and index options differ from patterns documented in mature markets. Rather than relying only on daily close-to-close data, future work can use intraday transaction and quote data to separate continuous volatility from discrete jumps and to evaluate whether realized-volatility measures improve short-maturity option pricing [19]. This direction is especially relevant for China because policy announcements, trading halts, and retail order flow may create concentrated intraday risk.

Second, machine-learning-assisted estimation can support model calibration without replacing economic structure. Neural networks or regularized learning methods can be used to identify regime shifts, select stable parameters, or approximate pricing functions when direct optimization is computationally expensive [20]. For China's option market, a useful application would be to combine no-arbitrage model restrictions with machine-learning tools that detect liquidity regimes and adjust calibration windows accordingly.

Third, rough-volatility and Hawkes-type jump extensions should be tested jointly. Rough-volatility models address persistent high-frequency volatility behavior, while Hawkes-type processes allow one jump to raise the likelihood of subsequent jumps [10,17]. A promising China-focused study could compare standard AJD, rough-volatility, and self-exciting-jump specifications using the same option panel, then evaluate not only pricing errors but also hedging performance and parameter stability.

5. Conclusion

This review has traced the evolution of option pricing theory from the elegant but restrictive Black-Scholes model to more flexible stochastic-volatility, jump-diffusion, SVJ, and AJD frameworks. The central pattern is clear: each new generation of models emerged because the previous generation failed to capture an empirically important market feature. Jumps address tail events, stochastic volatility addresses volatility clustering, and affine transform methods make richer models computationally feasible.

Nevertheless, greater model richness does not eliminate practical risk. The same features that improve pricing accuracy can also increase parameter instability, overfitting risk, and hedging difficulty. This tension is especially important in emerging markets, where liquidity conditions, retail participation, policy shocks, and clustered jump behavior may differ from those in mature markets. Future research should therefore move beyond importing mature-market specifications and should test China-specific combinations of high-frequency calibration, machine-learning-assisted estimation, rough volatility, and self-exciting jumps. Such work would better connect theoretical elegance with real-world market frictions.

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Conflicts of Interest

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