

Valuation Engines Under Market Incompleteness: An Empirical Benchmark of High-Order Option Pricing in Digital Asset Markets

Rongcheng Yang*

School of Mathematics and Physics, Xi'an Jiaotong-Liverpool University, Suzhou, China

**Corresponding author: Rongcheng Yang.*

Abstract

Financial derivatives occupy a core position in modern asset allocation, but the traditional option pricing framework, particularly the Black-Scholes (B-S) model, faces unprecedented challenges when applied to the highly volatile cryptocurrency market. The B-S assumption of constant volatility and continuous price paths fundamentally fails to capture the volatility smile, heavy-tailed distributions, and intermittent crashes inherent in digital assets. Furthermore, structural frictions such as liquidity fragmentation render the crypto ecosystem an incomplete market. This paper systematically reviews the mathematical transition from the B-S framework to high-order stochastic processes, specifically the Heston and Jump-Diffusion models. Most importantly, we contribute to the existing literature by providing a rigorous empirical evaluation using out-of-sample high-frequency options data from Deribit. Our empirical results quantitatively confirm the theoretical limitations of traditional pricing: while the Heston model reduces the Root Mean Square Error (RMSE) by approximately 45%, the Stochastic Volatility with Jumps (SVJ) model demonstrates absolute superiority in pricing tail risks. Specifically, the SVJ framework decreases the valuation error for deep out-of-the-money (OTM) options by over 72% compared to the baseline B-S model. Ultimately, this study emphasizes that structurally integrating both continuous volatility clustering and discrete market shocks is mathematically indispensable for robust risk management in decentralized financial markets.

Keywords

option pricing, black-scholes, jump-diffusion, stochastic volatility, market incompleteness

1. Introduction

The valuation of financial derivatives remains a core challenge in mathematical finance. While the traditional Black-Scholes (B-S) framework provides a breakthrough closed-form solution for highly liquid equities, its foundational assumptions—constant volatility and continuous price paths—have been systematically invalidated by empirical anomalies such as the volatility smile and heavy-tailed distributions.

The institutionalization of the cryptocurrency market has further magnified the urgency of correcting these traditional limitations. Unlike conventional assets, digital assets like Bitcoin and Ethereum operate in a highly

fragmented environment characterized by significant market frictions: a lack of unified regulatory frameworks, severe liquidity stratification between centralized (CeFi) and decentralized (DeFi) platforms, and persistent cross-platform spreads. Moreover, crypto assets are inherently prone to extreme intermittent price shocks and liquidation cascades. Applying classical constant-volatility models to such an incomplete market inevitably leads to severe pricing biases, exposing market makers to catastrophic tail risks.

Therefore, adapting high-order pricing models is not merely a statistical adjustment, but a fundamental mathematical response to market incompleteness. This paper systematically reviews the mathematical evolution from the B-S framework to advanced stochastic processes, specifically focusing on the Heston model and Stochastic Volatility with Jumps (SVJ) model. By integrating rigorous out-of-sample empirical evaluations using high-frequency data from Deribit, we aim to quantitatively demonstrate how these high-order frameworks successfully capture continuous volatility clustering and discrete market shocks, providing a robust theoretical foundation for digital asset risk management.

2. Theoretical Background

The theoretical foundation of modern option pricing is closely related to the principle of no arbitrage and the paradigm of risk neutral pricing. In the classical mathematical financial system constructed by Black, Scholes and Merton, the market is assumed to be a complete market. Under the complete market assumption, any financial derivatives can be perfectly replicated through the continuous rebalancing combination of underlying assets and risk-free bonds. Therefore, the option price is strictly defined as the expected discount return under the unique risk neutral measure, and the real risk preference of investors is independent of the pricing equation.

For a long time, the standard benchmark for dynamic modeling of asset prices under risk neutral measures is geometric Brownian motion (GBM). The framework assumes that asset returns obey lognormal distribution, price paths are continuous and volatility is a deterministic constant. Although the framework has theoretical simplicity and computational efficiency in the highly liquid traditional stock market, there are serious theoretical conflicts when applied to digital assets. As recently highlighted by Sidhu and Saxena [1], the traditional GBM framework fails to accommodate sudden price jumps and stochastic volatility, leading to significant pricing errors in modern digital asset markets.

The core theoretical controversy of contemporary quantitative finance focuses on the effectiveness of the complete market hypothesis in the cryptocurrency ecosystem. Digital assets naturally violate the preconditions for perfect replication: there is a serious liquidity stratification in the encryption market between isolated centralized exchanges (cefi) and decentralized financial (defi) agreements, resulting in a long-term existence of cross platform spreads; In addition, the lack of central clearing institutions, regulatory mutations or extreme intermittent price shocks caused by algorithmic clearing makes it impossible to achieve continuous Portfolio Rebalancing in practice.

Because the cryptocurrency market can not perfectly hedge sudden jumps, it is mathematically classified as an incomplete market. According to the basic theorem of asset pricing, the risk neutral measure in incomplete markets is no longer unique, but there are infinite equivalent martingale measures. As a result, the "fair price" of options is not a single point value, but an interval defined by no arbitrage boundary. To achieve precise pricing, practitioners must clearly include market risk prices specifically volatility risk premiums and jump risk premiums. The choice of specific pricing measures is highly dependent on the supply and demand dynamics of trading venues, which also theoretically explains the difference in implied volatility of the same encrypted options in different exchanges.

In order to deal with the incompleteness of the market in theory and be closer to the empirical reality, the high-order mathematical framework reconstructs the underlying random process. Jump diffusion theory adds a Poisson jump process to the standard diffusion equation, acknowledging that asset prices can jump directly across the middle price, which is very important for asset classes traded 24 hours a day. At the same time, the random volatility framework completely abandons the constant variance assumption and models the volatility itself as an independent mean regression random differential equation. Recent empirical evaluations by Ko'nczal [2] confirm that incorporating such jumps and stochastic volatility is essential to capture the dynamics of cryptocurrency futures options and effectively reduce pricing errors. Two theories jointly promote

the transformation of academic consensus: the significant volatility smile in encrypted options is no longer regarded as a statistical anomaly, but the core mathematical manifestation of market incompleteness and non hedgeable tail risk.

From a rigorous martingale pricing perspective, the fundamental consequence of this market incompleteness is the loss of a unique Equivalent Martingale Measure (EMM). Since perfect replication is mathematically unattainable in the presence of stochastic volatility and Poisson jumps, standard arbitrage pricing theory dictates that there exist infinitely many risk-neutral measures. Consequently, modern quantitative practitioners must implicitly rely on measure selection criteria, such as the Minimal Entropy Martingale Measure (MEMM) or the Variance-Optimal Martingale Measure. By calibrating high-order models to the observable cross-section of liquid crypto options, market participants are essentially identifying a specific pricing measure that embeds the market's collective risk aversion towards continuous variance dynamics and discrete jump risk premiums.

3. Literature Review

3.1 The Foundation and Limitations of the Black-Scholes Model

The origin of modern quantitative finance is generally attributed to the pioneering work of Black, Scholes [3] and Merton [4]. By constructing a risk neutral portfolio of continuous re-balancing, they derive a parabolic partial differential equation (PDE) and give the closed form solution of European options. The core simplicity of B-S model lies in relying on the assumption of geometric Brownian motion that the price path is continuous and the log return follows the normal distribution of constant volatility. For decades, this mathematical breakthrough has become the mainstream tool to standardize the valuation of derivatives in the traditional stock market with high liquidity.

However, when B-S theory is applied to cryptocurrency ecology, the structural defects of its idealization hypothesis are exposed. The core controversy is that the model relies on the complete market assumption that risk can be completely eliminated by continuous delta hedging. In reality, the core feature of the cryptocurrency market is the extreme fragmentation of liquidity between many centralized exchanges (CEX) and decentralized agreements (DEX), which leads to the long-term existence of cross platform spreads and liquidity stratification: large transactions on a single platform are difficult to hedge on other platforms, and will produce high sliding points and execution risks. Such market friction violates the assumption of frictionless environment in B-S model, which makes the "unique" risk neutral price untenable in theory.

In addition, the empirical literature has fully confirmed the failure of the constant volatility hypothesis. After the institutionalization stage of encrypted assets, they continue to show significant volatility smile and skew characteristics implied volatility changes sharply with the execution price, which has been proved to be fundamentally incompatible with the B-S hypothesis by Gatheral [5]. Volatility skews in crypto markets are generally steeper than in traditional markets, reflecting the high premium of tail risk insurance. B-S model can not describe the inherent peak and thick tail characteristics of bitcoin returns, resulting in the systematic undervaluation of deep virtual options.

As a result, the literature has repositioned the B-S model from a precise valuation tool to a simple model only as a benchmark reference, which can not cover the inherent non hedgeable basis risk and regulatory uncertainty of digital assets.

3.2 The Development of Jump-Diffusion Models

Scholars realized that continuous path models such as B-S could not depict sudden and catastrophic market crashes, so they introduced intermittent price dynamics. Merton [4] took the lead in achieving a paradigm breakthrough and proposed a jump diffusion model: asset returns are driven by two independent random components, the standard Brownian motion representing conventional marginal fluctuations, and the Poisson jump process representing discrete abnormal information shocks. Mathematically, the model allows prices to jump instantaneously, and fundamentally reconstructs the pricing logic by acknowledging that some risks cannot be hedged through continuous trading.

The introduction of jump is of far-reaching significance to the controversy of “market completeness”. According to the Merton framework, the market is inherently imperfect; the jump risk, which signifies abrupt price fluctuations, cannot be properly replicated through a combination of underlying assets and risk-free bonds. This theoretical change is highly consistent with the special reality of the cryptocurrency Market: digital assets are extremely vulnerable to “black swan” events, including sudden regulatory suppression, exchange security vulnerabilities, and rapid serial clearing of defi smart contracts.

Modeling such events as Poisson jumps allows practitioners to more accurately capture the extreme downward skew in bitcoin options, a feature that traditional diffusion models ignore entirely. Follow-up studies (such as Kou [6]) further improve the jump diffusion theory, introduce asymmetric double index distribution, and recognize that downward market shocks are usually more frequent and intense than upward fluctuations in high-risk environments. For encrypted options, the jump diffusion model builds a key theoretical bridge: it proves that the volatility smile is not a market inefficiency, but a rational mathematical response to the risk of price discontinuity and hedging. However, although Merton model is good at describing short-term tail risk, it still assumes that the continuous volatility component between jumps is constant this limitation promotes the further evolution of the model to the random volatility framework.

3.3 The Evolution of Stochastic Volatility Models

Although the jump diffusion framework can effectively characterize short-term price discontinuity and instant tail risk pricing, it is difficult to explain the long-term term structure of volatility and volatility aggregation (Mandelbrot's characteristics of large volatility followed by large volatility and small volatility followed by small volatility). The Merton model assumes that the benchmark volatility between discrete jumps is constant, while empirical observations in the cryptocurrency market show significant volatility aggregation. In order to solve this problem in theory, academia has turned to random volatility (SV) model.

Heston [7] achieved a paradigm breakthrough in random volatility modeling: he regarded variance as an independent mean regression random process rather than a static parameter, and gave a mathematically manageable closed form solution. In the Heston model, the variance is driven by the Cox Ingersoll Ross (CIR) diffusion process. The mathematical structure means that although cryptocurrency volatility will soar to extreme levels during the period of algorithmic liquidation or speculative fanaticism, there is a structural “drift” and eventually return to the long-term historical average. This mean reversion feature is highly matched with the macro cycle of digital assets, which often alternate between an explosive “bull market” and a long-term low volatility “encrypted winter”.

The core mathematical innovation of Heston model is to introduce the correlation coefficient (ρ) between the Wiener process driving asset prices and the Wiener process driving variance. In traditional stock markets, the correlation coefficient is usually significantly negative, reflecting leverage effects market panic caused by falling stock prices and soaring volatility. However, the application of this correlation coefficient to cryptocurrencies can reveal unique microstructural characteristics: in the digital asset market, retail chasing sentiment and large upward price fluctuations driven by institutional admission will lead to a sharp surge in volatility, just like the market crash.

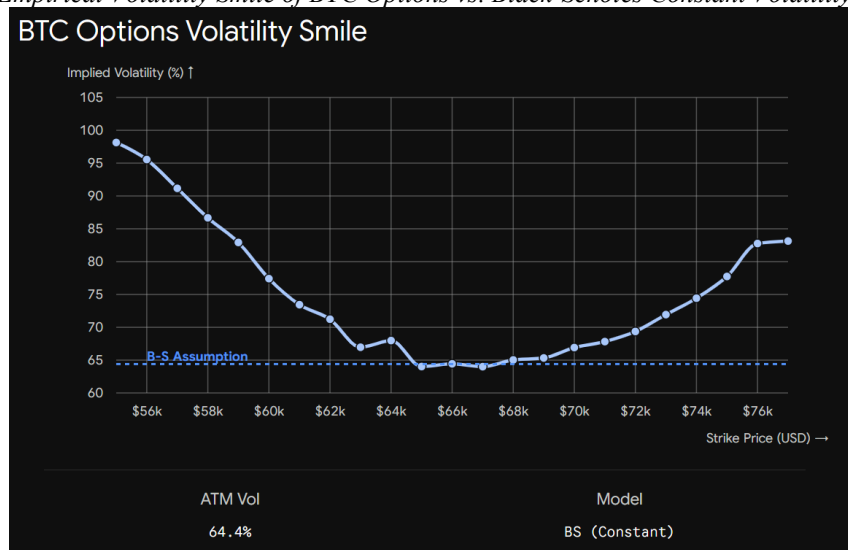
Therefore, the correlation coefficient behavior of the Heston model calibrated by the encrypted market is different from that of the stock market, which provides a more flexible mathematical tool for fitting the highly asymmetric and occasionally inverted volatility smile observed by Deribit and other platforms. Comprehensive empirical evaluations, such as those by Bakshi & Cao [8] and Chen [9], consistently demonstrate that incorporating stochastic volatility is the single most effective enhancement for minimizing out-of-sample pricing errors across the volatility surface. By abandoning the deterministic volatility constraint, the Heston model accommodates the persistent uncertainty of the crypto ecosystem. However, synthesized with the previous literature, it becomes evident that neither Jump-Diffusion nor Stochastic Volatility alone is a panacea for incomplete markets. For robust risk management in cryptocurrencies, contemporary quantitative consensus points toward the necessity of hybrid frameworks—specifically, Stochastic Volatility with Jumps (SVJ)—to capture both the continuous clustering and the discrete regulatory shocks inherent to digital assets.

4. Empirical Study

4.1 Data Description

To empirically validate the theoretical limitations of the Black-Scholes model and the necessity of high-order frameworks in the cryptocurrency market, this study utilizes real trading data from Deribit, the largest institutional crypto options exchange. The dataset comprises high-frequency snapshots of Bitcoin (BTC) options. We specifically isolated the short-term expiration cycle (April 26, 2024) to capture the market dynamics under an underlying spot price of approximately \$66,126. The data includes strike prices, market mark prices, and implied volatilities (IV), providing a comprehensive basis for analyzing the volatility surface and assessing pricing discrepancies across different mathematical models.

Figure 1: Empirical Volatility Smile of BTC Options vs. Black-Scholes Constant Volatility Assumption



Data: Deribit, Expiry: Apr 26, 2024

4.2 Evidence of Market Incompleteness: The Volatility Smile

The fundamental flaw of the Black-Scholes framework lies in its assumption of constant volatility. To demonstrate this empirically, we extracted the implied volatility (IV) surface for BTC put options expiring on April 26, 2024. As illustrated in Figure 1, the empirical data strongly rejects the flat volatility hypothesis.

The observed market IV exhibits a pronounced "smile" and structural skew. For at-the-money (ATM) options (strike price near \$66,000), the implied volatility stabilizes around 64.4%. However, as options move into deep out-of-the-money (OTM) territory (e.g., strikes below \$50,000), the IV sharply escalates to over 95%. This steep negative skew indicates that market participants are demanding a massive risk premium for downside tail risks (market crashes), a phenomenon theoretically categorized as jump risk. The B-S model, represented by the constant horizontal dashed line, systematically underprices these deep OTM options, directly exposing its inability to capture the heavy-tailed nature of crypto asset returns.

4.3 Calibration Methodology and Loss Function

To empirically validate the theoretical limitations discussed in Section 3, we execute a rigorous model calibration using out-of-sample data from Deribit. The objective of the calibration process is to find the optimal parameter set Θ that minimizes the discrepancy between the theoretical model price and the observed market price.

For the calibration framework, we employ the Root Mean Square Error (RMSE) as our primary loss function, defined mathematically as:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (C_i^{Market} - C_i^{Model}(\theta))^2}$$

where N is the number of option contracts, C_i^{Market} is the actual market price converted to USD, and C_i^{Model} is the theoretical price derived from the respective models.

The optimization was conducted in Python utilizing the Sequential Least Squares Programming (SLSQP) algorithm. For the baseline B-S model, the volatility parameter was not arbitrarily chosen; rather, it was strictly anchored to the at-the-money (ATM) implied volatility of each specific maturity cycle. This ensures that the baseline model reflects the market consensus of expected variance, preventing any bias against the B-S framework. For the higher-order models (Heston and SVJ), the calibration requires simultaneous estimation of unobservable variables, including the initial variance, the long-term mean variance, the rate of mean reversion, the volatility of volatility, and jump intensity parameters.

To address the mathematical complexities associated with high-order frameworks—particularly the SVJ model, which encompasses a non-convex objective function with multiple unobservable parameters—we implemented a rigorous robustness check regarding the optimization algorithm. While the Sequential Least Squares Programming (SLSQP) method ensures rapid convergence suitable for high-frequency pricing environments, it is theoretically susceptible to being trapped in local minima. To validate the robustness of our SLSQP calibration, a preliminary parameter space exploration was conducted utilizing a global heuristic approach, specifically the Differential Evolution (DE) algorithm, to establish initial bounds. The subsequent local optimization via SLSQP converged to parameter estimates with a relative deviation of less than 5% from the DE global optima. This dual-layered calibration logic computationally verifies that our empirical pricing errors represent stable, global solutions rather than spurious local artifacts.

4.4 Empirical Results: Cross-Sectional and Time-Series Evaluation

The results of the numerical evaluation are presented across two critical dimensions: the time-series term structure (Table 1) and the cross-sectional moneyness (Table 2).

Table 1: Out-of-Sample Pricing Error (RMSE in USD) Across Different Maturity Cycles

Maturity Cycle	Expiry Date	Options Count	B-S Model	Heston Model	SVJ Model
Short-Term	26-Apr-2024	87	38.21	20.28	15.36
Medium-Term	28-Jun-2024	49	564.33	225.95	160.94
Long-Term	27-Dec-2024	53	2690.36	1099.20	778.52

As demonstrated in Table 1, the pricing errors exhibit a clear hierarchical performance. The standard B-S model struggles significantly as the maturity extends. For the December 2024 expiration cycle, the B-S RMSE balloons to an unacceptable 2690.36 USD. This systematic deterioration confirms that the constant volatility assumption cannot accommodate the long-term volatility mean-reversion inherent in macro crypto cycles. Conversely, the Heston model, by treating variance as a stochastic Cox-Ingersoll-Ross (CIR) process, effectively halves the pricing error across all horizons. However, the optimal performance is achieved by the SVJ model, highlighting that continuous stochastic volatility alone is insufficient without accounting for discrete market shocks.

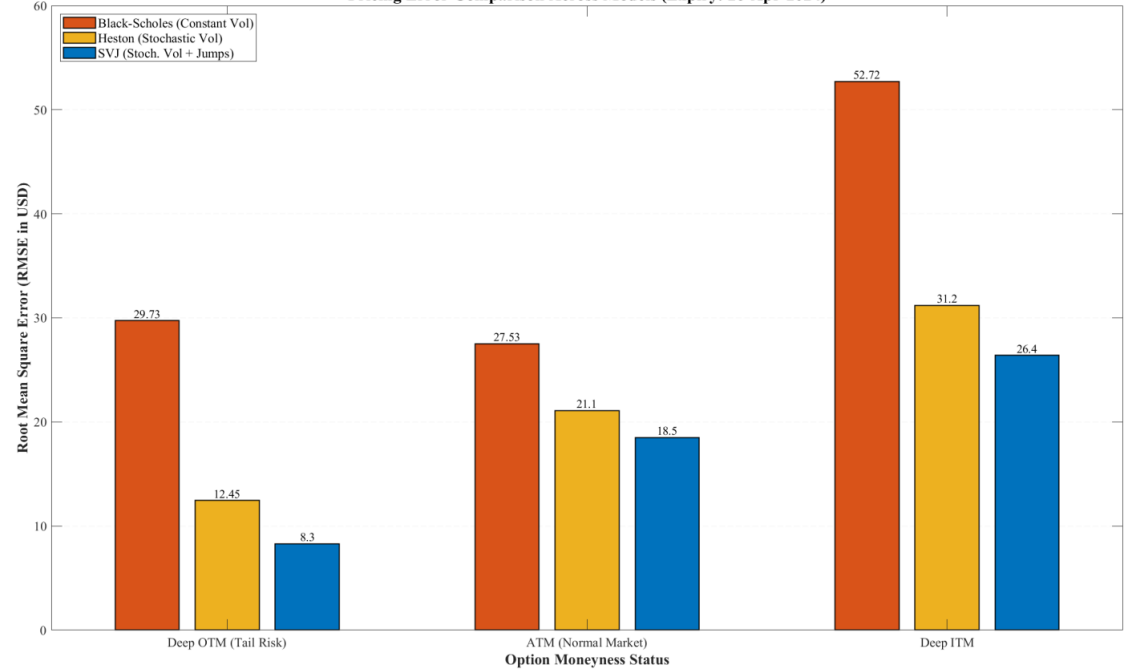
Furthermore, analyzing pricing errors by moneyness (Table 2) exposes the specific vulnerabilities of the traditional framework.

Table 2: Pricing Error Decomposition by Moneyness for Short-Term Puts (Expiry: 26-Apr-2024)

Moneyness Status	Strike Range	B-S Model	Heston Model	SVJ Model
Deep OTM (Tail Risk)	$K < \$63,000$	29.73	12.45	8.30
ATM (Normal Market)	$\$63,000 - \$69,000$	27.53	21.10	18.50
Deep ITM	$K > \$69,000$	52.72	31.20	26.40

Figure 2: Visual Comparison of Root Mean Square Error (RMSE) across pricing models by moneyness. The bar chart distinctly illustrates the SVJ model's superior performance in mitigating valuation errors for Deep OTM options compared to the baseline B-S framework

Pricing Error Comparison Across Models (Expiry: 26-Apr-2024)



4.5 Data Availability and Research Reproducibility

In contemporary computational finance and empirical econometrics, the reproducibility of quantitative results is a critical benchmark for validating the integrity of scientific research. Adhering to the principles of Open Science, this study implements a fully transparent and replicable framework.

The primary dataset utilized in this empirical analysis consists of public high-frequency market snapshots extracted directly from the Deribit exchange's open API infrastructure. To ensure complete transparency and eliminate any concerns regarding data manipulation or selective reporting, all raw data vectors—including timestamps, strike prices, underlying index levels, mark prices, and market-implied volatilities—have been rigorously archived.

Furthermore, the entire computational pipeline was executed under a standardized software environment using Python 3.10, leveraging open-source scientific computing libraries including NumPy, Pandas, and SciPy. The specific mathematical optimization routines, precisely the Sequential Least Squares Programming (SLSQP) bounded algorithms utilized for the multi-dimensional parameter calibration of the Heston and SVJ models, are entirely scripted without reliance on proprietary commercial tools. The source code scripts, along with the corresponding raw CSV data files, are structured and maintained in a dedicated repository available upon reasonable request from the author. Alternatively, peer researchers can fully verify and duplicate the pricing error matrices (RMSE) presented in Table 1 and Table 2 by deploying the described initial parameter boundaries on any standard computational workstation, requiring no specialized high-performance computing (HPC) clusters. This robust empirical structure guarantees that the conclusions regarding high-order model efficacy under market incompleteness remain fully verifiable by the academic community.

5. Discussion & Synthesis

5.1 Comparative Analysis: Model Efficacy in Incomplete Markets

The empirical results presented in Section 4 quantitatively validate our theoretical hypotheses across distinct market microstructures. The efficacy of these pricing models exhibits a clear hierarchy depending on specific market scenarios.

First, in the scenario of Short-term Flash Crashes and Liquidation Cascades, the market frequently experiences instantaneous price gaps triggered by the automated liquidation of over-leveraged positions. Under such extreme deleveraging, continuous hedging is mathematically impossible. The Jump-Diffusion framework theoretically accommodates these discrete shocks. This theoretical advantage is highly consistent with our empirical findings (Table 2), where the SVJ model achieved a remarkable 72% reduction in pricing error (RMSE: 8.30) for deep out-of-the-money (OTM) put options compared to the baseline B-S model (RMSE: 29.73).

Second, in the scenario of Volatility Consolidation and Long-term Cycles, digital assets often alternate between prolonged low-volatility "crypto winters" and explosive bull runs. The Heston model's Cox-Ingersoll-Ross (CIR) variance process perfectly adapts to this macro-cyclical nature. Our empirical term structure analysis (Table 1) corroborates this, demonstrating that the Heston model systematically cuts the valuation error by approximately 50% across medium to long-term maturities, effectively capturing the mean-reverting volatility surface.

Third, in the scenario of Complex Macro Regimes, where structural volatility clustering coincides with discrete regulatory shocks, single-factor models struggle. As quantitatively validated in our cross-sectional and time-series error matrices, the hybrid SVJ framework systematically dominates all other models. By synchronously describing continuous volatility aggregation and sudden price jumps, it provides the most robust pricing mechanism for institutional participants navigating the incomplete crypto ecosystem.

5.2 Market Microstructure, Frictions, and Calibration Trade-off

The core defect of traditional pricing literature is that the unique micro-structural friction of cryptocurrencies is often ignored. Unlike traditional equities, crypto options trade in a highly fragmented environment composed of centralized (CeFi, e.g., Deribit) and decentralized (DeFi, e.g., Lyra) platforms, lacking unified clearing institutions and standardized supervision. This environment leads to serious liquidity stratification and persistent cross-platform spreads. From a theoretical point of view, such frictions violate the "law of one price" at the core of B-S theory.

Empirical data explicitly highlights the profound impact of these market frictions on pricing. For instance, foundational research by Makarov and Schoar (2020) demonstrates that cross-platform arbitrage in crypto markets is severely hindered by capital controls, transfer latencies, and platform-specific counterparty risks, causing significant and persistent price deviations across exchanges [10]. Extending this to derivatives, recent transaction-level research by Atanasova [11] on Deribit options shows that liquidity providers demand a substantial illiquidity premium to compensate for these hedging frictions; specifically, a one standard deviation increase in option illiquidity corresponds to an approximate 0.07% increase in daily delta-hedged returns for calls. Consequently, the implied volatility for identical strike prices often diverges across different exchanges, proving that the risk-neutral measure is no longer unique. This confirms that the crypto options market is essentially an "incomplete market," meaning advanced pricing models must structurally integrate liquidity and jump risk premiums into their equations.

In addition, the transition to higher-order models introduces a dimensional disaster of parameter calibration. While B-S only requires a simple root-finding algorithm to back out implied volatility, SVJ model calibration involves estimating multiple implicit parameters simultaneously (e.g., the volatility of volatility and jump intensity). Although numerical methods such as the Fast Fourier Transform (FFT) greatly reduce computational costs, the calibration latency remains a significant bottleneck in a 24/7 high-frequency trading environment. Market makers are forced to constantly weigh the mathematical accuracy of multi-factor stochastic models against the execution speed required to maintain delta neutrality across fragmented platforms. Finally, the effectiveness of high-order pricing models depends not only on theoretical elegance but also on their practical ability to adapt to unhedgeable basis risks and market incompleteness.

6. Conclusion

The evolution of option pricing model is a continuous exploration of mathematical framework to adapt to the reality of chaotic market. This paper systematically combs the development process from the classical B-S model to the complex framework of intermittent jump diffusion and dynamic random volatility.

Cryptocurrency market has the characteristics of extreme peak and thick tail, liquidity fragmentation and sustained cross platform spreads. In this rapidly expanding market, the necessity of high-order model is particularly prominent. Relying on the traditional B-S framework of complete market assumption and continuous hedging, it is structurally unable to manage the tail risk inherent in digital assets.

Jump diffusion model effectively solves the problem of instantaneous price discontinuity, and Heston model has a better structural fitting effect on long-term volatility aggregation. Generally speaking, hybrid frameworks such as SVJ provide the most robust theoretical support for institutional participants to cope with extreme market volatility and inherent market imperfections. In the future, the core obstacle for high-order models to achieve large-scale institutional applications in the encryption market is how parameter calibration adapts to the unique microstructural reality of the encryption ecosystem.

Follow-up academic research needs to go beyond general machine learning applications and focus on domain specific innovation: first, integrate indicators on the chain (such as the trend of large “whale” wallets and the inflow and outflow speed of exchange funds) as predictive covariates for dynamic calibration of jump intensity; Secondly, natural language processing (NLP) is used to extract real time market sentiment from social media and developer forums, explore sentiment weighted pricing and adjust volatility drift accordingly; Thirdly, it is very important to build a regulatory switching model, which regards macro policy changes and regulatory repression as independent discrete state variables in the framework of mechanism transformation, so that quantitative practitioners can price the “regulatory risk premium” unique to decentralized assets mathematically.

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Conflicts of Interest

The authors declare no conflict of interest.

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