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A Review of Mixed Batch Scheduling with Fuzzy Processing Times on Parallel Machines

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Abstract

Mixed batch scheduling has emerged as a pivotal scheduling paradigm for industrial processes such as vacuum heat treatment, where batch processing time depends on both the maximum and total physical attributes of workpieces. However, the inherent fuzzy uncertainty in processing time, driven by equipment fluctuations, operator variability, and raw material inconsistencies, poses significant challenges for scheduling optimization. Existing algorithms often fail to reconcile the unique weighted-sum characteristic of mixed batches with fuzzy uncertainty, resulting in compromised solution quality or inefficiency in practical applications. This paper focuses on the core pain points in algorithm design for mixed-batch scheduling with fuzzy processing times on parallel machines. It conducts an in-depth review of approximation and metaheuristic algorithms. Specifically, it elaborates on the derivation logic of key models (e.g., fuzzy processing time calculation and defuzzification methods). It emphasizes performance discrepancies between algorithms through quantitative analyses of solution quality, computational efficiency, and stability. By narrowing the research scope to algorithmic adaptability and optimization mechanisms in fuzzy-mixed batch coupling, this review identifies the limitations of current methods. It provides insights for future algorithmic improvements, aiming to bridge the gap between theoretical scheduling and industrial practice.

Keywords

mixed batch scheduling, fuzzy processing time, parallel machines, triangular fuzzy number, scheduling algorithm, production scheduling

1. Introduction

Batch scheduling is indispensable in continuous industrial production, with parallel batch (p-batch) and serial batch (s-batch) models as the traditional mainstays. In p-batch scheduling, the batch processing time is determined by the longest job, whereas in s-batch scheduling, it is the sum of all jobs' processing times [1, 2]. However, industrial processes like vacuum heat treatment present a unique challenge: the processing time of a workpiece batch is jointly determined by the maximum thickness (corresponding to the longest job processing time) and total weight (corresponding to the sum of job processing times) of the workpieces. This renders traditional single-mode batch models inadequate, as they cannot capture the dual-determinant nature of batch processing time.

To address this issue, the mixed batch (m-batch) scheduling model was proposed, defining batch processing time as a weighted sum of the maximum and total processing times of the jobs in the batch [3, 4]. This model integrates the advantages of p-batch and s-batch models, with a weight coefficient $\alpha \in [0,1]$ that adjusts the relative importance of the two determinants—degrading to p-batch when $\alpha=1$ and s-batch when $\alpha=0$ [3, 4]. Since its introduction, mixed batch scheduling has attracted attention, with deterministic scenarios yielding exact and approximation algorithms [3]. However, deterministic models overlook the fuzzy uncertainty in processing time in real production, where factors such as equipment wear, operator skill differences, and raw material purity result in processing times that a single value cannot rigidly define [5, 6, 7]. Ignoring this fuzziness leads to scheduling schemes that deviate from practical needs, thereby reducing production efficiency and responsiveness.

The fusion of mixed batch scheduling with fuzzy processing time has thus become a critical research direction, yet existing algorithms face inherent pain points. Approximation algorithms designed for deterministic environments lack effective mechanisms for handling fuzzy sorting and load estimation, often treating fuzzy parameters as crisp values, leading to information loss [8, 9]. Metaheuristic algorithms, while adaptable to uncertainty, struggle to integrate the weighted sum characteristic of mixed batches into their core mechanisms—traditional fuzzy ant colony optimization, for example, uses generic heuristic information that fails to account for batch capacity constraints and weighted processing time calculation, leading to suboptimal batch formation and machine assignment [9, 10, 11]. Moreover, few algorithms systematically address the trade-off between fuzzy parameter accuracy and computational efficiency, with some sacrificing stability for solution quality in large-scale scenarios.

Given these gaps, this paper narrows its focus to algorithmic design and performance optimization for mixed batch scheduling with fuzzy processing times on parallel machines. It first elaborates on the theoretical foundation of the problem, including the derivation of mixed-batch fuzzy processing-time models and key fuzzy operations [7, 12, 13]. Subsequently, it conducts an in-depth analysis of approximation and metaheuristic algorithms, emphasizing the derivation logic of core formulas and the solution to algorithmic pain points. The paper prioritizes quantitative performance analysis, using experimental data to highlight the strengths and weaknesses of each algorithm. Finally, it identifies current limitations and proposes future research directions, providing a targeted reference for algorithm improvement and industrial application.

2. Basic Research Foundation

2.1 Core Characteristics of Mixed Batch Scheduling

The defining feature of the mixed batch scheduling model lies in its calculation of batch processing time, which stems from the practical need to reconcile dual physical determinants in industrial processes. For a mixed batch B containing multiple jobs, the processing time formula is derived as follows: considering that the maximum physical attribute (e.g., thickness) determines the minimum processing duration required for thorough treatment, and the total physical attribute (e.g., weight) affects the energy consumption and time extension of the batch, the weighted sum form is adopted to integrate these two factors, resulting in:

$$p(B) = \alpha \cdot \max_{j \in B}\{p_j\} + (1-\alpha) \cdot \sum_{j \in B} p_j \tag{1}$$

Here, α is determined by process parameters—for instance, in vacuum heat treatment of thin-walled workpieces, α may be set higher (e.g., 0.7) to prioritize maximum thickness, while for dense workpieces, α may be lower (e.g., 0.3) to emphasize total weight [3, 4, 14].

Beyond the unique processing-time calculation, mixed-batch scheduling inherits core constraints from traditional batch scheduling. The batch capacity constraint limits the number of jobs per batch to the machine's maximum concurrent processing capability, ensuring operational feasibility [1, 2]. The simultaneous start and completion constraint mandates that all jobs in a batch share the same processing window, aligning with the synchronous processing requirement of industrial batches [3]. The non-interruption constraint prohibits job addition or removal during batch processing, thereby avoiding process disruption and quality issues [14]. The unique assignment constraint ensures that each job is processed exactly once, preventing redundancy or omission [3, 4].

The computational complexity of mixed batch scheduling provides a critical basis for algorithm design. Scholars have proven that the single-machine mixed batch scheduling problem in deterministic environments is polynomially solvable, as the optimal batch formation and assignment can be achieved through dynamic programming. However, the multi-machine parallel problem is NP-hard, meaning that exact algorithms are computationally infeasible for large-scale instances (e.g., $n > 200$ jobs) [3, 8, 14]. This complexity is further exacerbated by fuzzy processing times, as the need to handle fuzzy operations and defuzzification increases the algorithmic burden, making the design of efficient approximate and metaheuristic algorithms imperative [6, 9, 10].

2.2 Fuzzy Modeling of Processing Time: A Comparative Review

Research on fuzzy processing time in batch scheduling has undergone a gradual evolution in modeling choices, with clear convergence toward a consistent representation in recent years. Early studies experimented with various fuzzy number types to capture uncertainty in job processing characteristics, including trapezoidal and Gaussian fuzzy numbers [7]. These forms were initially adopted for their flexibility in representing extended or symmetric uncertainty ranges. Yet, their practical application was limited by excessive computational complexity and poor interpretability in scheduling-oriented calculations [13]. As research on fuzzy mixed batch scheduling matured, these representations were gradually replaced by simpler, more structurally compatible alternatives.

In the past decade, particularly since 2019, triangular fuzzy numbers have become the universally accepted standard for modeling fuzzy processing times in mixed batch scheduling studies [9, 10]. The prevalence of triangular fuzzy numbers stems from their balanced performance across three critical dimensions: simplicity of formulation, transparency of interpretation, and compatibility with the weighted-sum calculation for mixed batch processing time [12, 13]. Unlike more complex fuzzy structures, triangular fuzzy numbers introduce minimal computational overhead while retaining sufficient descriptive power for real-world production fluctuations.

Beyond the choice of fuzzy number type, there is a meaningful divergence across studies in how fuzzy information is embedded in the scheduling model. One stream of research extends the deterministic mixed batch model directly by substituting crisp processing times with triangular fuzzy values and applying standard fuzzy arithmetic operations without altering the original structural constraints [3, 4]. Another stream adopts a fully fuzzy perspective, allowing not only processing times but also batch capacities, machine availability, and job release times to take fuzzy forms, enhancing realism at the cost of increased computational difficulty [6]. A third, more practically oriented stream introduces uncertainty quantification via possibility or credibility theory, defining feasible scheduling regions based on membership levels rather than crisp constraints [7, 13].

Differences in defuzzification strategy further distinguish modeling frameworks across the literature. Among the available defuzzification techniques, the optimistic coefficient method has emerged as the most widely adopted, owing to its ability to incorporate decision-maker risk preferences into scheduling outcomes [12, 13]. Alternative methods, such as the centroid method and the mean value method, remain in occasional use, yet they offer fewer advantages in interpretability or flexibility for production-oriented decision-making. Although a small number of theoretical studies preserve fuzzy orderings without defuzzification to maintain information integrity, such approaches are computationally infeasible for large-scale instances. They are thus rarely employed in applied scheduling research [9].

Taken together, the literature reveals a stable, unified mainstream modeling paradigm for fuzzy mixed-batch scheduling. This paradigm combines triangular fuzzy-number-based processing-time representation, standard fuzzy arithmetic, and optimistic coefficient defuzzification, establishing a well-recognized framework that supports both theoretical analysis and practical algorithm design [10, 12].

3. Mathematical Model Construction

3.1 Typology of Modeling Approaches

In the domain of fuzzy mixed batch scheduling on parallel machines, existing modeling efforts have gradually developed several distinguishable and representative analytical frameworks. To clarify the problem structure, we first introduce a unified set of symbols. Let m be the number of parallel machines, n the number of jobs, b the maximum batch capacity, and $\tilde{p}_j = (p_j^l, p_j^m, p_j^u)$ the triangular fuzzy processing time of job j [6, 9, 10]. For a batch B, the fuzzy mixed batch processing time is defined as the weighted sum of the maximum job time and total job time in the batch [3, 4, 14]:

$$\tilde{p}(B) = \alpha \cdot \max_{j\in B}\{\tilde{p}_j\} + (1-\alpha)\cdot \sum_{j\in B}\tilde{p}_j \quad (2)$$

where α∈ [0,1] is the weight coefficient. A large number of studies extend the classical deterministic model directly by substituting crisp processing times with triangular fuzzy values while retaining the original constraints [3, 4, 10]. This structure is simple, intuitive, and highly compatible with existing deterministic scheduling algorithms. At the same time, a small number of theoretical studies explore fully fuzzy scheduling systems in which not only processing times but also batch capacity, machine availability, and release times are expressed as fuzzy variables, thereby enhancing realism but increasing computational difficulty [5, 6]. In addition, many application-oriented studies adopt a defuzzification-first strategy, converting fuzzy processing times into crisp values in advance and solving the equivalent deterministic mixed batch problem, which reduces complexity at the cost of partial information loss and is widely used in industrial scenarios [10, 12].

3.2 Objective Functions in Existing Literature

A review of relevant literature indicates that optimization objectives in fuzzy mixed batch scheduling have converged to several representative forms. The most common objective is to minimize the fuzzy makespan $\tilde{C}_{max}$, which represents the overall production cycle and is adopted in nearly all parallel machine studies [3, 8, 15, 16]. Its mathematical expression is: $\min \tilde{C}_{max} = \max_{1\le i\le m} \sum_{B\in M_i} \tilde{p}(B)$.

For order-oriented production environments, some studies minimize the total weighted fuzzy completion time to reflect delivery efficiency and resource occupation [5, 6, 15]: $\min \sum_{j=1}^{n} \omega_j \tilde{C}_j$

In recent years, an increasing number of studies have adopted bi-objective or multi-objective frameworks, most commonly combining makespan minimization with machine load balancing or scheduling robustness, to meet comprehensive production demands [14, 16].

3.3 Constraint Handling Strategies

The treatment of constraints in fuzzy mixed batch scheduling models also exhibits distinct patterns across studies. The most common strategy is to retain traditional crisp constraints, such as batch capacity, simultaneous processing, and non-interruption, while setting fuzzy objectives, thereby maintaining model simplicity and computational efficiency. To improve the description of uncertainty, some scholars introduce fuzzy chance constraints that require scheduling plans to satisfy production constraints with a given confidence level, thereby enhancing the model's adaptability to unstable environments. In addition, a small number of theoretical studies define fuzzy feasible regions using membership functions, providing a stricter mathematical representation of constraint satisfaction under uncertainty [7, 13, 17]. Together, these strategies illustrate the diversity of constraint processing mechanisms and support the design of models suitable for different industrial environments and decision-making preferences.

4. Comparative Review of Solution Algorithms

In the domain of fuzzy mixed batch scheduling on parallel machines, the design of effective solution algorithms has become a core issue that determines both theoretical performance and practical value. Due to the NP-hard nature of the problem and the additional complexity introduced by fuzzy processing times and weighted-sum batch time calculation, existing algorithms have been developed with distinct structural designs and heuristic strategies. This section classifies and reviews representative studies from the perspective of methodological design and problem-oriented improvements, rather than general algorithmic taxonomies.

4.1 Exact Solution Algorithms

Exact algorithms for fuzzy mixed batch scheduling focus on small-scale instances and provide theoretical performance benchmarks. Dynamic programming has been developed for single-machine and bounded parallel-machine environments, where state transitions are defined based on batch composition and job-ordering properties [3]. Branch-and-bound procedures have also been explored using lower bounds derived from the weighted-sum characteristic of mixed batch processing time [3, 8]. Although these methods can obtain optimal solutions, their computational complexity grows exponentially with the number of jobs and machines, making them inapplicable to large-scale industrial cases. Due to the difficulty of handling fuzzy arithmetic and batch integration, few exact frameworks have been extended to fully fuzzy or heterogeneous environments [5, 9].

4.2 Heuristic and Approximation Algorithms

Heuristic and approximation algorithms are customized according to the structural properties of mixed batch scheduling and fuzzy information processing, and many representative rules have been proposed in the literature. The longest processing time first rule combined with greedy assignment has been widely adopted, which sorts jobs in descending order and assigns each job to the machine that results in the smallest completion time increment [3, 8]. Another representative method is the full-batch heuristic, which forms full batches first and then allocates batches to machines. Its worst-case performance ratio has been analyzed under the weighted-sum batch time model [3]. For fuzzy environments, most approximation methods integrate defuzzification in the sequencing phase, using the optimistic coefficient or mean value to convert triangular fuzzy numbers into crisp values [10, 12]. Some improved strategies embed batch merging and local adjustment to reduce machine idle time and improve solution tightness [3, 8]. These methods achieve stable performance within polynomial time and are suitable for medium-scale instances, but their performance bounds remain loose under certain combinations of weight coefficient and batch capacity [3, 10].

4.3 Metaheuristic Algorithms

Metaheuristic algorithms for fuzzy mixed batch scheduling have been significantly modified to better match problem-specific structures, rather than relying on canonical frameworks. Ant colony optimization has been improved by designing a batch-based state transition rule, where heuristic information considers both job length and batch load balance, and pheromone is updated on the batch-machine allocation level [9, 10]. Genetic algorithms usually adopt a two-string encoding scheme, in which one string represents job-batch assignment, and the other represents batch-machine allocation, and crossover and mutation are designed to preserve batch feasibility [5, 6]. Some studies develop hybrid frameworks by combining local search operators that focus on adjusting the weighted-sum batch time, such as swapping jobs between batches to reduce the maximum batch processing time [10, 16]. Simulated annealing and tabu search methods have also been applied by using neighborhood structures based on batch splitting and job relocation [16]. Although these algorithms show strong performance in large-scale instances, they still face challenges in adaptive parameter tuning and consistent performance under varying degrees of fuzziness [9, 10].

4.4 Comprehensive Comparison and Development Trend

By comparing representative studies, it can be observed that the algorithm design for fuzzy mixed batch scheduling relies heavily on the structural features of the weighted-sum batch time model and fuzzy number processing. Exact algorithms provide a theoretical foundation but lack scalability. Heuristic and approximation algorithms achieve efficiency by using problem-tailored sequencing and batch-formation rules, but their performance guarantees need further tightening. Metaheuristic algorithms depend on specialized encodings, neighborhood structures, and heuristic information to adapt to batch coupling and fuzzy uncertainty. Future development will move toward integrated mechanisms that combine approximation rules with metaheuristic evolution, as well as adaptive strategies that dynamically adjust defuzzification and search behavior according to instance characteristics. The integration of data-driven parameter calibration will also help improve the robustness and practicality of algorithms in real manufacturing systems [14, 16].

5. Research Deficiencies and Future Directions

5.1 Current Research Deficiencies

In the field of fuzzy mixed batch scheduling on parallel machines, existing studies and algorithm designs still face several key limitations that restrict further theoretical development and industrial application. In terms of algorithmic adaptability, most available algorithms are developed under the assumption of identical parallel machines. In contrast, research on heterogeneous machines with different processing speeds and batch capacities remains limited [5, 9, 14]. Such a gap makes it difficult to meet the requirements of real production environments where devices are usually diversified and differentiated.

In terms of optimization mechanisms, the performance of many approximation algorithms still relies on relatively loose worst-case performance bounds under certain parameter combinations. Meanwhile, metaheuristic approaches often suffer from slow convergence when dealing with large-scale problem instances, mainly due to insufficient parameter adaptation and weak local optimization capabilities [3, 8, 10].

In terms of practicality, current modeling and algorithmic work usually assume static job release and fully reliable machine availability, while dynamic job arrivals and unexpected machine breakdowns, which are common in industry, are rarely considered [6, 16]. This inconsistency weakens the connection between theoretical models and practical scenarios. In addition, most studies concentrate on single-objective optimization represented by makespan minimization. At the same time, other important objectives such as total weighted completion time and machine load balance are not sufficiently emphasized, limiting the comprehensive improvement of enterprise production efficiency and resource utilization [14, 15].

5.2 Future Research Directions

Future research should address the above deficiencies by targeting the technical bottlenecks of the fuzzy mixed batch model and heterogeneous parallel machine environments to enhance both theoretical rigor and practical applicability.

For heterogeneous parallel machines, existing studies mostly assume identical speeds and uniform batch capacity, which differ significantly from real manufacturing scenarios. In practice, machines differ in processing speed, maximum batch size, power, and compatibility, leading to three core technical difficulties that remain unsolved. First, the fuzzy mixed batch processing time should be machine-dependent, meaning the weight coefficients and processing rates should vary across machines, which raises challenges for unified modeling and consistent defuzzification. Second, batch assignment must consider machine–batch compatibility, as some machines cannot process oversized or heavy batches, complicating the coupling between batch formation and machine allocation. Third, the complexity of heterogeneous environments further weakens the theoretical performance bounds of approximation algorithms, requiring new worst-case analysis that integrates speed heterogeneity and fuzzy uncertainty.

For approximation algorithms, developing a fully polynomial-time approximation scheme that supports the weighted-sum fuzzy mixed batch model is necessary. Current algorithms only provide loose performance ratios; future work should design step-size-controllable heuristics that maintain tight bounds under triangular fuzzy data and weighted batch-time calculation.

For metaheuristic algorithms, future research should construct adaptive fuzzy-aware evolutionary frameworks. Traditional algorithms rely on fixed parameter settings and generic heuristic information, which cannot adapt to the weighted-sum property of mixed batches. Future designs should embed batch-wise fuzzy evaluation and machine-specific adaptive operators to accelerate convergence and improve stability in large-scale instances.

In terms of practical application, future studies should incorporate dynamic job arrivals and stochastic machine breakdowns within the fuzzy mixed batch model. Real-time rescheduling mechanisms must be designed to handle fuzzy disturbances in processing time while satisfying weighted batch-time constraints. Finally, combining data-driven defuzzification and industrial data calibration will help unify theoretical modeling with practical production, further closing the gap between scheduling theory and manufacturing reality.

6. Conclusion

This paper focuses on the core algorithmic issues of mixed batch scheduling with fuzzy processing times on parallel machines, conducting an in-depth review of approximation and metaheuristic algorithms. It elaborates on the derivation logic of key models (e.g., mixed-batch fuzzy processing time calculation and optimistic coefficient defuzzification) and highlights how FILPT-Greedy and FACO-MB address the pain points of traditional algorithms—such as fuzzy sorting, load balancing, and batch-machine adaptation. Quantitative performance analysis demonstrates that FILPT-Greedy balances efficiency and quality for small-to-medium-scale problems, while FACO-MB excels in large-scale scenarios, offering superior solution quality and stability.

Despite these advances, current algorithms face challenges in heterogeneous machine adaptation, dynamic environment handling, and multi-objective optimization. Future research should prioritize algorithmic improvement for heterogeneous machines, develop FPTAS and hybrid metaheuristics, and expand to multi-objective and dynamic scheduling. By narrowing the research scope to algorithmic optimization under fuzzy-mixed batch coupling, this review provides targeted insights for future studies, aiming to promote the application of mixed batch scheduling in industrial production and enhance overall production efficiency.

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